Predictive Power of Strong Coupling in Theories with Large Distance Modified Gravity

Gia Dvali*

Center for Cosmology and Particle Physics,

Department of Physics, New York University

4 Washington Place, New York, NY 10003

Abstract

We consider theories that modify gravity at cosmological distances, and show that any such theory must exhibit a *strong coupling phenomenon*, or else it is either inconsistent or is already ruled out by the solar system observations. We show that all the ghost-free theories that modify dynamics of spin-2 graviton on asymptotically flat backgrounds, automatically have this property. Due to the strong coupling effect, modification of the gravitational force is source-dependent, and for lighter sources sets in at shorter distances. This universal feature makes modified gravity theories predictive and potentially testable not only by cosmological observations, but also by precision gravitational measurements at scales much shorter than the current cosmological horizon. We give a simple parametrization of consistent large distance modified gravity theories and their predicted deviations from the Einsteinian metric near the gravitating sources.

*email: gd23@nyu.edu

1 Introduction

The observed accelerated expansion of the Universe [1] could result [2] from a breakdown of the standard laws of gravity at very large distances, in particular, caused by extra dimensions becoming gravitationally-accessible at distances of order the current cosmic horizon [3].

In this study, we shall formulate some very general properties of large distance modified gravity theories, that follow from the consistency requirements based on 4D effective field theory considerations. These arguments allow to give a simple parametrization and display some very general *necessary* properties of such theories, even without knowing their complete explicit form.

In this work, we shall be interested in theories that, *i)* are ghost free, *ii)* admit the weak field linearized expansion on asymptotically flat backgrounds, and *iii)* modify Newtonian dynamics at a certain crossover scale, r_c. Motivated by cosmological considerations one is tempted (and usually has to) to take r_c of the order of the current cosmological horizon $H_0^{-1} \sim 10^{28}$cm, but in most of our discussions we shall keep r_c as a free parameter.

There exists an unique class of ghost-free linearized theories of a spin-2 field with the above properties. These theories have a tensorial structure of the Pauli-Fierz massive graviton, and are described by the following equation

$$\mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} \; - \; m^2(\Box)\,(h_{\mu\nu} \; - \; \eta_{\mu\nu} h) \; = \; T_{\mu\nu}. \tag{1.1}$$

Here,

$$\mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} \; = \; \Box h_{\mu\nu} \; - \; \Box \eta_{\mu\nu} h \; - \; \partial^\alpha \partial_\mu h_{\alpha\nu} \; - \; \partial^\alpha \partial_\nu h_{\alpha\mu} \; + \; \eta_{\mu\nu} \partial^\alpha \partial^\beta h_{\alpha\beta} \; + \; \partial_\mu \partial_\nu h \tag{1.2}$$

is the linearized Einstein's tensor, and as usual $h \equiv \eta_{\alpha\beta} h^{\alpha\beta}$. $T_{\mu\nu}$ is a conserved source. The gravitational coupling constant $(8\pi G_N)$ is set equal to one, for simplicity. The difference of the above theory from the Pauli-Fierz massive graviton is, that in our case $m^2(\Box)$ is not a constant, but a more general operator, that depends on $\Box$. In order to modify Newtonian dynamics at scale r_c, $m^2(\Box)$ must dominate over the first Einsteinian term at the scales $r \gg r_c$, or equivalently for the momenta $k \ll r_c^{-1}$. This implies that to the leading order

$$m^2(\Box) \simeq r_c^{2(\alpha\,-\,1)} \Box^\alpha \,, \tag{1.3}$$

with $\alpha \, < \, 1$.

The class of theories of interest thus can be parameterized by a single continuous parameter α. As we shall see in a moment, unitarity further restricts α from below by demanding $\alpha \, \geqslant \, 0$. The limiting case $\alpha \, = \, 0$ corresponds to the massive Pauli-Fierz graviton.

Currently, the only theory that has a known ghost-free generally-covariant non-linear completion is $\alpha \, = \, 1/2$, which is a five-dimensional brane world model (DGP-model) [3]. The limiting case $\alpha = 0$ for massive graviton,[1] unfortunately, has no known consistent non-linear completion, and it is very likely that such completions are impossible [7, 8], at least in theories with finite number of states.

The non-linear completions for theories with other values of α have not been studied. It is not the purpose of the present study to discover such completions. Instead, we shall display some necessary conditions that such completions (if they exist) must satisfy. As we shall see, the effective field theory considerations combined with consistency assumptions give us a powerful tool for predicting some of the general properties of the theories, even without knowing all the details of the underlying dynamics.

The central role in our consideration is played by the *strong coupling* phenomenon of extra graviton polarizations, discovered in [9] both for massive gravity ($\alpha = 0$) and for [3] ($\alpha = 1/2$). We shall show, that this phenomenon must be exhibited by theories with arbitrary values of α from the unitarity range $0 \leqslant \alpha \leqslant 1$. Due to this strong coupling, a gravitating source of mass M, on top of the ordinary Schwarzschild radius $r_g \equiv 2G_N M$, acquires a second (intermediate) physical distance $r_g \leqslant r_* \leqslant r_c$, which for extra graviton polarizations plays the role somewhat similar to the Schwarzschild horizon. The new scale r_* depends both on r_g and r_c, and thus, varies from source to source.

The above fact predicts the following modification of the gravitational potential for an arbitrary source localized within r_*-radius

$$\delta \sim \left(\frac{r}{r_c}\right)^{2(1-\alpha)} \sqrt{\frac{r}{r_g}}. \tag{1.4}$$

Because of this predictive power, the theories of modified gravity are very restrictive and potentially testable by precision gravitational measurements. Our analysis complements and solidifies the earlier study of [10, 11], in which the corrections (1.4) where already suggested.

Notice, that for the cosmologically interesting case $r_c = H_0^{-1}$, the entire observable Universe is the only source for which $r_g = H_0^{-1} = r_*$. Therefore, the concept of r_* is crucial for understanding stability (absence of ghosts or tachions) of cosmological backgrounds, thus, invalidating many perturbative approaches, as the sources of size r_* have to be studied nonperturbatively.

[1]Note, $\alpha = 0$ is not necessarily a massive gravity, since subleading corrections to $m^2(\Box)$ can further distinguish the theories. For instance, it is likely [4] that higher dimensional generalizations [5, 6] of [3], should also correspond to $\alpha = 0$ case.

2 Unitarity Constraints

We shall first derive the unitarity lower bound on α. For this, consider a one-graviton exchange amplitude between the two conserved sources ($T_{\mu\nu}$ and $T'_{\mu\nu}$)

$$(\text{Amplitude})_h \propto \frac{T'_{\mu\nu}T^{\mu\nu} - \frac{1}{3}T^{\mu}_{\mu}T'^{\nu}_{\nu}}{\Box - m^2(\Box)}. \tag{2.1}$$

The scalar part of the (Euclidean) graviton propagator then has the following form

$$\Delta(k^2) = \frac{1}{k^2 + m^2(k^2)}. \tag{2.2}$$

As said above, modification of gravity at large distances implies that the denominator is dominated by the second term, for $k \to 0$. To see how unitarity constrains such a behavior, let the spectral representation of the propagator $\Delta(k^2)$ be

$$\Delta(k^2) = \int_0^{\infty} ds\, \frac{\rho(s)}{k^2 + s}, \tag{2.3}$$

where $\rho(s)$ is a bounded spectral function. The absence of the negative norm states demands that $\rho(s)$ be a semi-positive definite function. Evaluating (2.3) for the small k and using parameterization (1.3), we get

$$\frac{1}{r_c^{2(\alpha-1)}k^{2\alpha}} = \int_0^{\infty} ds\, \frac{\rho(s)}{k^2 + s}. \tag{2.4}$$

Non-negativity of $\rho(s)$ implies that α cannot be negative. In the opposite case, $\Delta(k^2)$ would be zero for $k^2 = 0$, which is impossible for non-negative $\rho(s)$. Thus, from unitarity, it follows that $\alpha = 0$ is the lowest possible bound.

In general, branch cuts in the propagator reflect the existence of the continuum of states, which could be interpreted as the signature of extra dimension. Thus, in a sense, large distance modification of gravity leads to the existence of extra non-compact dimensions, as in [3].

3 Extra Polarizations

A graviton satisfying the equation (1.1), just as in the Palui-Fierz case, propagates five degrees of freedom. These include, two tensor, two vector and one scalar polarizations. The extra polarizations (especially the scalar) play the central role in the strong coupling phenomenon, and it is therefore useful to separate the 'new' states from the 'old' (massless spin-2) states.

We shall now integrate the extra three polarizations out and write down the effective equation for the two remaining tensorial ones. For this we shall first rewrite equation (1.1) in the manifestly gauge invariant form, using the Stückelberg method. We can rewrite $h_{\mu\nu}$ in the following form

$$h_{\mu\nu} = \hat{h}_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu, \tag{3.1}$$

where the Stückelberg field A_μ is the massive vector that carries two extra helicity-1 polarizations. The remaining extra scalar state resides partially in A_μ and partially in $\hat{h}_{\mu\nu}$. Written in terms of $\hat{h}_{\mu\nu}$ and A_μ,

$$\mathcal{E}_{\mu\nu}^{\alpha\beta}\hat{h}_{\alpha\beta} - m^2(\Box)(\hat{h}_{\mu\nu} - \eta_{\mu\nu}\hat{h} + \partial_\mu A_\nu + \partial_\nu A_\mu - 2\eta_{\mu\nu}\partial^\alpha A_\alpha) = T_{\mu\nu}, \tag{3.2}$$

the equation (1.1) becomes manifestly invariant under the following gauge transformation

$$\hat{h}_{\mu\nu} \to \hat{h}_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad A_\mu \to A_\mu - \xi_\mu, \tag{3.3}$$

where ξ_μ is the gauge parameter. Note that the first, Einstein's term is unchanged under the replacement (3.1), due to it's gauge invariance. We now wish to integrate out A_μ through its equation of motion,

$$\partial^\mu F_{\mu\nu} = -\partial^\mu(\hat{h}_{\mu\nu} - \eta_{\mu\nu}\hat{h}), \tag{3.4}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. Before solving for A_μ, note that by taking a divergence from the equation (3.4) we get the following constraint on $\hat{h}_{\mu\nu}$

$$\partial^\mu\partial^\nu\hat{h}_{\mu\nu} - \Box\hat{h} = 0, \tag{3.5}$$

which means that $\hat{h}_{\mu\nu}$ is representable in the form

$$\hat{h}_{\mu\nu} = \tilde{h}_{\mu\nu} + \eta_{\mu\nu}\frac{1}{3}\Pi_{\alpha\beta}\tilde{h}^{\alpha\beta}, \tag{3.6}$$

where $\Pi_{\alpha\beta} = \frac{\partial_\alpha\partial_\beta}{\Box} - \eta_{\alpha\beta}$ is the transverse projector. $\tilde{h}_{\mu\nu}$ carries two degrees of freedom. Notice that, since the last term in (3.6) is gauge invariant, under the gauge transformations (3.3), $\tilde{h}_{\mu\nu}$ shifts in the same way as $\hat{h}_{\mu\nu}$ does.

Coming back to the equation (3.4) and solving it for A_μ we get

$$A_\nu = -\frac{1}{\Box}\partial^\mu(\hat{h}_{\mu\nu} - \eta_{\mu\nu}\hat{h}) - \partial_\nu\Theta, \tag{3.7}$$

where Θ is arbitrary. The gauge invariance demands that under the transformations (3.3), Θ shifts in the following way

$$\Theta \to \Theta + \frac{1}{\Box}\partial_\alpha\xi^\alpha. \tag{3.8}$$

Substituting (3.7) back to (3.2), expressing $\hat{h}_{\mu\nu}$ in terms of $\tilde{h}_{\mu\nu}$ through (3.6), and choosing the gauge appropriately, we can write the resulting effective equation for $\tilde{h}_{\mu\nu}$ in the following form

$$\left(1 - \frac{m^2(\Box)}{\Box}\right)\mathcal{E}_{\mu\nu}^{\alpha\beta}\tilde{h}_{\alpha\beta} = T_{\mu\nu}, \tag{3.9}$$

which has the tensorial structure identical to the linearized Einstein's equation. This latter fact indicates that $\tilde{h}_{\mu\nu}$ indeed propagates only two tensor polarizations, characteristic for the massless graviton.

Because $\tilde{h}_{\mu\nu}$ propagates only two degrees of freedom, the one-particle exchange amplitude between the two conserved sources $T_{\mu\nu}, T'_{\mu\nu}$ mediated by $\tilde{h}$ is

$$(\mathrm{Amplitude})_{\tilde{h}} \propto \frac{T'_{\mu\nu}T^{\mu\nu} - \frac{1}{2}T^{\mu}_{\mu}T'^{\nu}_{\nu}}{\Box - m^2(\Box)}, \tag{3.10}$$

which continuously recovers the massless graviton result in the limit $m^2 \to 0$. This fact however *does not* avoid the well known van Dam-Veltman-Zakharov (vDVZ) discontinuity [12], because (3.10) is only a part of the full one-particle exchange amplitude. This becomes immediately clear if we notice that the metric excitation that couples to the conserved $T_{\mu\nu}$ is $\hat{h}_{\mu\nu}$ (or equivalently $h_{\mu\nu}$), which depends on $\tilde{h}_{\mu\nu}$ through (3.6). The full physical amplitude is generated by the latter combination of $\tilde{h}_{\mu\nu}$ and is equal to (2.1), which clearly exhibits the vDVZ-type discontinuity in the $m^2(\Box) \to 0$ limit.

4 Strong Coupling of Extra States and Concept of r_*

We have seen that in the considered class of theories graviton should contain three extra 'longitudinal' polarizations, which lead to vDVZ discontinuity in linearized theory. Hence, any theory of modified gravity that remains weakly coupled at the solar system distances, is automatically ruled out by the existing gravitational data.

Thus, the only point that could save such theories is the breakdown of the linearized approximation at the solar system distances, that is, the *strong coupling phenomenon* [9]. In the other words, the only theories of modified gravity that can be compatible with observations are the strongly coupled ones. Any attempt of curing the strong coupling, will simply rule out the theory.

Fortunately, the same extra polarization that creates a worrisome discontinuity in the gravitational amplitude, also provides the strong coupling which invalidates the unwanted result at solar system scales. We shall now generalize the results of [9] to theories with arbitrary α.

Consider a theory of graviton that in the linearized approximation satisfies the equation (1.1). We shall assume that this theory has a generally-covariant non-linear completion. The non-linear completion of the first term is the usual Einstein's tensor. The completion of the second term is unknown for general α. In fact, extrapolation from the only known case of $\alpha = 1/2$ [3] tells us that if a completion exists, it will probably require going beyond four-dimensions. Discovering such a completion is beyond our program. In fact, knowing the explicit form of this completion is unnecessary for our analysis, provided it meets consistency requirements listed above. Then, we will be able to derive general properties of the strong coupling and the resulting predictions.

The propagator for the graviton satisfying equation (1.1) has the following form,

$$D_{\mu\nu;\alpha\beta} \,=\, \left(\frac{1}{2}\,\tilde{\eta}_{\mu\alpha}\tilde{\eta}_{\nu\beta} + \frac{1}{2}\,\tilde{\eta}_{\mu\beta}\tilde{\eta}_{\nu\alpha} - \frac{1}{3}\,\tilde{\eta}_{\mu\nu}\tilde{\eta}_{\alpha\beta}\right)\frac{1}{\Box\,-\,m^2(\Box)}\,, \tag{4.1}$$

where

$$\tilde{\eta}_{\mu\nu} \,\equiv\, \eta_{\mu\nu} \,-\, \frac{\partial_\mu\partial_\nu}{m^2(\Box)} \,=\, \eta_{\mu\nu} \,-\, r_c^{2(\alpha\,-\,1)}\frac{\partial_\mu\partial_\nu}{\Box^\alpha}. \tag{4.2}$$

The existence of the terms that are singular in r_c^{-1} is the most important fact. This singularity is precisely the source of the strong coupling observed in [9] for $\alpha = 0$ and $\alpha = 1/2$ cases. The terms that are singular in r_c^{-1} come from the additional, scalar polarization state of the resonance graviton. In the Stückelberg language given in (3.3), this scalar polarization resides partially in A_μ and partially in the trace of $\hat{h}_{\mu\nu}$. If we denote the canonically normalized longitudinal polarization by χ, then, ignoring the spin-1 helicity vector, the full metric fluctuation to the linear order can be represented as

$$h_{\mu\nu} \,=\, \tilde{h}_{\mu\nu} \,-\, \frac{1}{6}\eta_{\mu\nu}\chi \,+\, r_c^{2(1\,-\alpha\,)}\frac{\partial_\mu\partial_\nu}{3\Box^\alpha}\chi. \tag{4.3}$$

Notice that $\tilde{h}_{\mu\nu}$ is the same as in (3.6). The same state is responsible for the extra attraction, that provides factor of 1/3 in the one graviton exchange amplitude, as opposed to 1/2 in the standard gravity, leading to vDVZ discontinuity.

The strong coupling of the longitudinal gravitons, however, has a profound effect on the discontinuity. The effect of the longitudinal gravitons becomes suppressed near the gravitating sources, where the linearized approximation breaks down. Due to the strong coupling effects, the gravitating sources of mass M, on top of the usual Schwarzschild gravitational radius $r_g \equiv 2G_N\,M$, acquire the second physical radius, which we shall call r_*. Breakdown of the linearized approximation near gravitating sources, was first noticed in the context of $\alpha = 0$ theory in [13], and the underlying strong coupling dynamics was uncovered in [9]. The key point of the present discussion is that both the strong coupling, and the resulting r_*-scale is a property of theories with arbitrary α.

For the longitudinal graviton χ, the latter radius plays the role somewhat similar to the one played by r_g for the two usual tensor polarizations $\tilde{h}_{\mu\nu}$. Namely, due to the strong coupling, at $r = r_*$ the non-linear self interactions of χ become important, and the expansion in series of G_N (that is, expansion in r_g/r) breaks down.

The concept of r_* plays the central role in any large distance modified theory of gravity, as we shall now discuss. Consider a localized static gravitating source $T_{\mu\nu} \,=\, \delta^0_\mu\delta^0_\nu M\delta(r)$ of gravitational radius r_g. Then, sufficiently far from the source, the linearized approximation should be valid, and the metric created by the source can be found in one graviton exchange approximation to linear order in G_N,

$$h_{\mu\nu} = \frac{\delta^0_\mu \delta^0_\nu - \frac{1}{3}(\eta_{\mu\nu} - r_c^{2(1-\alpha)} \frac{\partial_\mu \partial_\nu}{\Box^\alpha})}{\Box - r_c^{2(\alpha-1)} \Box^\alpha} r_g \, 4\pi\delta(r). \tag{4.4}$$

The term in the numerator, that is singular in $1/r_c$, vanishes when convoluted with any conserved test source $T'_{\mu\nu}$, in accordance with (2.1). Hence, at the distances below r_c, the metric has an $\frac{r_g}{r}$ form, but with a *wrong* (scalar-tensor type) tensorial structure, manifesting vDVZ discontinuity. However, in nonlinear interactions, the singular in $1/r_c$ terms no longer vanish and in fact wash-out the linear effects, due to the strong coupling. The scale of the strong coupling can be figured out by generalizing the analysis of [9, 10]. The straightforward power counting then gives that the leading singularity in r_c is of the order of $r_c^{4(1-\alpha)}$, and comes from the trilinear interaction of the longitudinal gravitons. For $\alpha = 0$ this reproduces the result of [9] (see also [15]). For general α this vertex has a momentum dependence of the form

$$(r_c k)^{4(1-\alpha)} k^2. \tag{4.5}$$

Then, the scale r_* corresponds to a distance from the source for which the contribution from the above trilinear vertex, becomes as important as the linear one given by (4.4). It is obvious that the corresponding distance is given by

$$r_* = (r_c^{4(1-\alpha)} r_g)^{\frac{1}{1+4(1-\alpha)}}. \tag{4.6}$$

For distances $r \ll r_*$ the correction to the Einsteinian metric coming from the longitudinal gravitons is suppressed by powers of r_*. The leading behavior can be fixed from the two requirements. First, $\chi(r)$ should become of order r_g/r_* at $r = r_*$, in order to match the linear regime (4.4) outside the r_*-sphere. Secondly, the solution inside r_* must be possible to approximate by the analytic series in $r_c^{2(\alpha-1)}$. These two requirements fix the leading behavior as

$$\chi(r \ll r_*) \sim \frac{r_g}{r_*} \left(\frac{r}{r_*}\right)^{\frac{3}{2}-2\alpha}, \tag{4.7}$$

which gives the relative correction to the gravitational potential given by (1.4).

For $r_c = H_0^{-1}$, the predicted anomalous perihelion precession of a planetary orbit is given by

$$\text{Perihelion advance per orbit} \quad \sim (rH_0)^{2(1-\alpha)} \sqrt{\frac{r}{r_g}}, \tag{4.8}$$

where r_g is the gravitational radius of the binding star (or any other source), and r is the radius of the planetary orbit around it. This expression reproduces the result of [10].

As pointed out in the latter paper, for some interesting values of α these corrections are strong enough to be tested in precision gravitational measurement experiments. For $\alpha = 1/2$ theory this was also pointed out in [17], with the correction from the cosmological self-accelerated background [14] taken into the account.

In fact, existing Lunar Ranging constraints already rule out theories with α significantly above 1/2, and the new generation of the improved accuracy measurements [19], will probably test the $\alpha \simeq 1/2$ case.

Notice that for $\alpha = 1/2$, (1.4) matches the known explicit solutions derived both in $1/r_c$-expansion [16], as well as exactly [18].

5 Conclusions

Possibility of large-distance modification of gravity is a fundamental question, motivated by the Dark Energy Problem. In this paper we tried to stress the important role played by the *strong coupling phenomenon* in this class of theories. In particular, it was already appreciated [9], [11] that the strong coupling is the only cure that saves any such theory from being immediately ruled out by the solar system observations, due to the fact that any such ghost-free theory must contain an extra scalar polarization and is subject to vDVZ discontinuity at the lineareazed level. This fact makes impossible the existence of consistent weakly coupled theories of modified gravity (relevant for cosmic observations).

Interestingly, in the class of theories considered, the same scalar polarization that creates the problem at the linearized level, also invalidates it by exhibiting the strong coupling phenomenon.

Due to this strong coupling phenomenon, gravitating objects are 'endowed' with a physical scale r_*, which for the extra graviton polarizations plays the hole of a second 'horizon'. At distances $r_* \sim r$ the non-linear interactions of χ catch-up with the linear part, and expansion in terms of the Newtonian coupling constant can no longer be trusted. Hence, vDVZ-discontinuity is invalidated.

For scales $r \ll r_*$, the metric can be found in series of $\left(\frac{r}{r_c}\right)^{2(1-\alpha)}$ - expansion (or possibly exactly). The assumption that the true solution can be approximated in such analytic series, fixes the form of the leading correction to the Eisteinian metric in the form of (1.4).

This fact, makes theories of modified gravity potentially testable by the precision gravitational experiments in the solar system, and in particular by Lunar Ranging experiments.

Another important question is predicting the explicit form of modified Friedmann equation for generic α, in the spirit of [20, 21].

Note Added

In this note we wish to briefly comment on the issue of stability of some cosmological backgrounds, that are created by large distance modification of gravity, since this issue is governed by the concept of r_*, which is central to our work. For the phenomenologically most interesting case $r_c \sim H_0^{-1}$, all the gravitating sources, that fit within the present Hubble volume, are smaller than their own r_*-s. The only observable source that is of the size of its own r_*, is the Universe itself. This fact immediately explains why the smaller-than-H_0^{-1} sources gravitate almost normally, whereas the cosmological expansion of the Universe is strongly modified. In [3] the result of the latter modification is the appearance of the self-accelerated background [14], [2]. Lately, the perturbative stability of this background was subject of some discussion. Our analysis indicates that the issue can only be answered non-perturbatively. Indeed, since for the Universe $r_* \sim H_0^{-1}$, the perturbation theory breaks down. Another indication of this breakdown is that on the self-accelerated background $\Box\chi \sim H_0^2/M_P$, which the reader can easily check. This simple fact is hard to reconcile with the claims of [22], about the existence of ghost-type instabilities on this background, since these are based on the perturbative treatment, which is invalid. The detailed discussion of the recent article [23] points out yet another problem with the latter analysis. The key point being, that for understanding the dynamics of excitations, that are localized within their own r_*-s, again, the perturbative arguments cannot be trusted.

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